647510 **MSC4MatC404x** **Seat No :________**

M.Sc.(Maths.) Semester - 4 (*CBCS*) Examination
March/April - 2024 (NEW COURSE)
Graph Theory (Core)

Time: 02:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (A) Answer any TWO of the following questions. **[10]**

(1) Show that in a given transport network G, the value of flow w from the source s to sink t is less than or equal to the capacity of any cut separating s from t.

(2) Let G be a graph and it does not contain any self-loop. Suppose for any pair of vertices $u, v \in G$, there is a unique path between u and v in G. Prove that G is a tree.

(3) Let G be a graph and it contains exactly two odd vertices, say $x, y \in V(G)$. Prove that x and y both lies in the same component of G.

Q.1 (B) Answer any ONE of the following questions. **[04]**

(1) For a simple graph $G = (V, E)$ on n vertices, q edges and k components, show that $q \leq \frac{1}{2}(n-k)(n-k+1)$.

(2) Explain Maximal Flow in brief.

Q.2 (A) Answer any TWO of the following questions. **[10]**

(1) Let G be an acyclic graph with n vertices and k components. Prove that, G has $n-k$ edges.

(2) Show that a connected graph G is an Euler graph if and only if G can be decomposed into edge disjoint cycles.

(3) Explain Transport Network with example.

Q.2 (B) Answer any ONE of the following questions. **[04]**

(1) Define the following terms:

(i) Euler Graph (ii) Unicursal Graph

(iii) Hamiltonian Cycle (iv) Closure of a Graph

(2) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits.

Q.3 (A) Answer any TWO of the following questions. **[10]**

(1) A graph is a tree if and only if it is minimally connected.

(2) Let $G = (V, E)$ be a connected graph and S is a cut set for G. If F is a cycle in G, then show that $|E(F) \cap S|$ is even.

(3) Show that every tree has either one or two centres.

Q.3 (B) Answer any ONE of the following questions. **[04]**

(1) Define the following terms:

(i) Tree (ii) Pendant Vertex

(iii) Acyclic Graph (iv) Minimally Connected Graph

(2) Show that a graph G with n vertices, $n-1$ edges, and no circuits is connected.

Q.4 (A) Answer any TWO of the following questions. **[10]**

(1) Define Edge Connectivity and Vertex Connectivity. Prove the following:

i. The edge connectivity of a graph G cannot exceed the degree of the vertex with the smallest degree in G.

ii. The vertex connectivity of any graph G can never exceed the edge connectivity of G.

(2) Prove that Kuratowaski's first graph and second graph, both are non-planner graphs.

(3) Let $G = (V, E)$ be a simple connected graph with n vertices, e edges and f faces.

Then show that:

(i) $e \geq \frac{3f}{2}$ (ii) $e \leq 3n - 6$

Q.4 (B) Answer any ONE of the following questions. **[04]**

(1) Define Adjacency Matrix with an example.

(2) Show that any simple planar graph can be embedded in a plane such that every edge is drawn as a straight line segment.

Q.5 (A) Answer any TWO of the following questions. **[10]**

(1) Prove that every tree with at least two vertices is always 2-chromatic graph.

(2) Show that in a given transport network G, the value of flow w from source s to sink t is less than or equal to the capacity of any cut separating s from t.

(3) Show that a graph of n vertices is a complete graph if and only if its chromatic its polynomial is $P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \cdots (\lambda - n + 1)$.

Q.5 (B) Answer any ONE of the following questions. **[04]**

(1) Explain coloring of a graph with example.

(2) State and Prove *Max-Flow Min-Cut* Theorem.

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